

Systematic study of the singularity mechanism in heavy quarkonium decays

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We investigate in detail the role of heavy meson loops in the transition from $J^{PC} = 1^{--}$ sources to candidates for QCD “exotics”, such as $Z_c(3900)$, $Z_b(10610)$ and $Z'_b(10650)$. We demonstrate that, if a vector state strongly couples to a heavy meson pair in an S -wave and this system decays to another heavy meson pair (e.g. via pion emission), again in an S -wave, the pertinent diagrams get enhanced significantly, if the intermediate states are (near) on-shell and have small relative momenta. In a limited kinematic range this mechanism generates “singularity regions” that lead to the creation of a large number of low energy heavy meson pairs, providing an ideal environment for the formation of hadron-hadron bound states or resonances. For instance, we predict that the signals for Z_b and Z'_b should be a lot stronger in $\Upsilon(6S)$ decays due to this mechanism, if these states are indeed hadron-hadron resonances. The findings of this work should be valuable for deepening our understanding of the nature of the mentioned states.

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I. INTRODUCTION

The recent results from the BESIII [1] and Belle Collaboration [2] have attracted immediate attention from the hadron physics community. The observation of an enhancement $Z_c(3900)$ with charge in the invariant mass spectrum of $J/\psi\pi^\pm$ in $e^+e^- \rightarrow Y(4260) \rightarrow J/\psi\pi^+\pi^-$ with high statistics may be a clear evidence for QCD “exotics” in the charmonium energy region. The observation is also confirmed by the CLEO-c experiment in $e^+e^- \rightarrow \psi(4170) \rightarrow J/\psi\pi\pi$ in the invariant mass of $J/\psi\pi$ [3]. The mass of $Z_c(3900)$ is close to the $\bar{D}D^*$ threshold. It therefore is an interesting analogue to $Z_b(10610)$ and $Z_b(10650)$, located close to the $\bar{B}B^*$ and \bar{B}^*B^* thresholds, respectively, which were observed by the Belle Collaboration [4, 5] last year. There have been a lot of theoretical efforts on the interpretation of the Z_b states [6–16]. Almost immediately after BESIII published their data, different interpretations [17–29] were proposed for understanding the nature of the $Z_c(3900)$.

In Ref. [20] it was argued that, if there is a significant amount of $\bar{D}D_1 + c.c.$ in the wave function of $Y(4260)$, namely, if the $Y(4260)$ is predominantly of molecular nature, then a large number of low energy $L = 0$ $\bar{D}D^*$ pairs would be naturally produced, since both $\bar{D}D_1$ and $\bar{D}D^*$ can be nearly on-shell in a relative S -wave simultaneously. This leads to a significant enhancement of the pertinent loops and provides an ideal environment for the formation of $\bar{D}D^*$ bound or resonant systems. Such a kinematic condition is similar to the so-called “triangle singularity” discussed in Refs. [30, 31].

The very same scenario also unavoidably leads to the appearance of a cusp, i.e. a pronounced structure in the close vicinity of the S -wave threshold. In contrast to a resonance, however, there is no nearby pole present in the amplitude. In Ref. [20] it was argued that the location, strength and shape of the $Z_c(3900)$ signal are inconsistent with its interpretation as a cusp. Thus, an explicit resonance is needed in addition. Still, if the $Z_c(3900)$ qualifies as a \bar{D}^*D resonance, the mechanism described should still lead to a significantly enhanced production rate, since it naturally provides a large number of low-energy \bar{D}^*D pairs.

The two-cut condition is operative in a limited kinematic range only. As a result, the strength of the cusp as well as the number of low energy S -wave $\bar{D}D^*$ pairs available for the formation of the resonance, will strongly depend on the total energy of the system. We therefore predict that, if the $Z_c(3900)$ is a resonance produced via non-perturbative $\bar{D}D^*$ interactions (a $\bar{D}D^*$ resonance), its production rate should depend strongly on the total energy of the system. In other words, even in the absence of a pronounced cusp a hadron-hadron resonance can be produced, but in its presence the production of a hadron-hadron resonance should be largely enhanced. In this sense the total energy dependence of the $Z_c(3900)$ production rate can be regarded as a diagnostic tool for understanding its composition,

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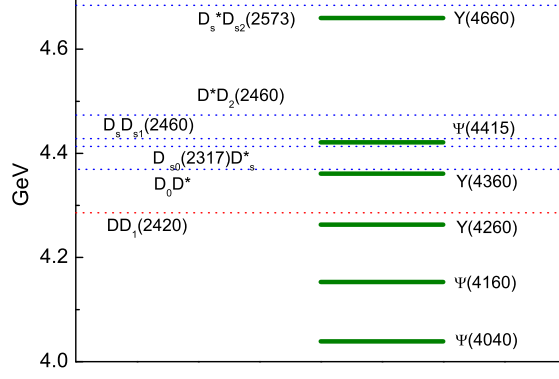


FIG. 1: The spectrum of vector charmonium and relative S -wave open charmed thresholds.

if the two cut-condition is really responsible for the copious production of $Z_c(3900)$ in the decay of $Y(4260)$. On the contrary, if the $Z_c(3900)$ is predominantly of tetra-quark nature, as proposed in Ref. [17], the dependence of the production rate on the total energy of the system should be much weaker. This prediction can be straightforwardly tested experimentally in e^+e^- annihilations.

In this work, we identify the relative S -wave heavy meson thresholds relevant for the decay of heavy vector mesons into a pion and the isovector system of interest and discuss the possible phenomenological implications of some of those in detail. In the end of the paper we will also discuss briefly P -wave thresholds. Our analysis should provide a path towards a better understanding of the structure of some potential QCD exotics.

II. ANALYSIS OF THE S -WAVE SINGULARITY MECHANISM

In the vector sector, the relative S -wave open charm thresholds are depicted in Fig. 1. Notice that the $\bar{D}D_1(2420)$ system provides the first relative S -wave open charm threshold. In addition, it is located near the mass position of $Y(4260)$. It was pointed out in Ref. [20] that if the $Y(4260)$ is dominated by a molecular $\bar{D}D_1$ component, one can understand the appearance of the $Z_c(3900)$ in $e^+e^- \rightarrow Y(4260) \rightarrow J/\psi\pi\pi$ quite naturally. In order to distinguish an explicit resonance from a cusp effect, besides looking at the particular shape and strength of the signal in the above mentioned reaction as done in Ref. [20], we here explore a broader kinematic region.

We stress that molecular states and hadron-hadron resonances cannot be formed by broad intermediate states [32]. In addition, a cusp effect will also become invisible with broad intermediate states [33]. Taking this into account, there is only limited number of thresholds that can produce significant cusp effects for relative S -wave low-momentum $\bar{D}D^*$ or $\bar{B}B^*$ pairs, i.e. $\bar{D}^{(*)}D_1(2420)$, $\bar{D}_s^*D_{s0}(2317)$, $\bar{D}_s^*D_{s1}(2460)$, $\bar{D}^*D_2(2460)$ and $\bar{D}_s^*D_{s2}(2573)$ in the charm sector, and $\bar{B}B_1$ and some other corresponding bottomed meson pairs in the bottom sector.

In order to demonstrate the dynamic features of the relative S -wave couplings and low-momentum $\bar{D}D^*$ scatterings, we employ the following Lagrangians in the calculation

$$\begin{aligned} \mathcal{L}_Y = & iy(D_a^\dagger Y^i \bar{D}_{1a}^{\dagger i} - D_{1a}^{\dagger i} Y^i \bar{D}_a^\dagger) \\ & + y\epsilon^{ijk}(D_{1a}^{\dagger i} Y^k \bar{D}_a^{\dagger j} - D_a^{\dagger j} Y^k \bar{D}_{1a}^{\dagger i}) + H.c. \end{aligned} \quad (1)$$

for the $Y(4260)$ coupling to other $D^{(*)}$ mesons, and

$$\begin{aligned} \mathcal{L}_{D_1} = & i\frac{h'}{f_\pi}[3D_{1a}^i(\partial^i\partial^j\phi_{ab})D_b^{\dagger j} - D_{1a}^i(\partial^j\partial^j\phi_{ab})D_b^{\dagger i} \\ & + 3\bar{D}_a^{\dagger i}(\partial^i\partial^j\phi_{ab})\bar{D}_{1b}^j - \bar{D}_a^{\dagger i}(\partial^j\partial^j\phi_{ab})\bar{D}_{1b}^i] + H.c. \end{aligned} \quad (2)$$

for the D_1 coupling to $D^{(*)}$ and a pion. Here the D (\bar{D}) field contains the annihilation operators for the $c\bar{q}$ ($\bar{c}q$) quark configuration. The D^* and D_1 fields are constructed analogously. To account for the heavy quark spin symmetry, D

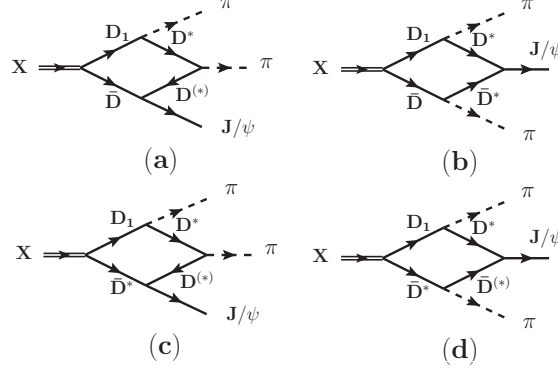


FIG. 2: Feynman diagrams demonstrating a vector meson X with hidden charm decays into $J/\psi\pi\pi$ via the singularity mechanism. The Feynman diagrams in the bottomonium sector are analogous.

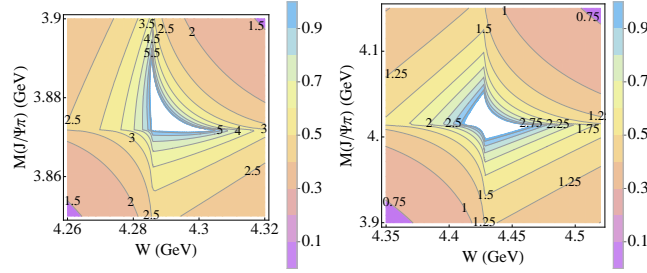


FIG. 3: The singularity region of DD^* (left panel) and D^*D^* (right panel) without considering the widths of the intermediate mesons. The charge conjugate transitions, which behave in a similar manner, are not included here. The numbers in the figures are the absolute values of three point scalar functions and the numbers in the sidebar are their relative strengths.

and D^* are collected into a single multiplet which makes them share the same coupling constants y and h' [34, 35]. The details for the other interactions can be found in Ref. [12].

In the cusp kinematic region where the intermediated states are (nearly) on shell, the exchanged charmed meson between the J/ψ and a pion is far off-shell. Within such a kinematic region the propagator for the exchanged charmed meson is approximately $1/M_{D^{(*)}}^2$ and the $\bar{D}^{(*)}D^* \rightarrow J/\psi\pi$ amplitude can be treated as a local function $\mathcal{F}(M(J/\psi\pi), t)$ with $M(J/\psi\pi)$ and t the invariant mass of $J/\psi\pi$ and t -channel momentum transfer, respectively. Since $\mathcal{F}(M(J/\psi\pi), t)$ does not vary drastically within the range of $M(J/\psi\pi)$ and t , the four-point loop function in Fig. 2 can be expressed as the following typical expression and be analyzed as a three-point function:

$$\begin{aligned}
 M &= \int \frac{d^4l}{(2\pi)^4} \frac{G\epsilon_X^i \epsilon_{J/\psi}^j (3q_1^i q_1^j - |q_1|^2 \delta^{ij}) \mathcal{F}(M(J/\psi\pi), t)}{(l^0 - \frac{|\vec{l}|^2}{2m_{D_1}} + i\varepsilon)(p^0 - l^0 - \frac{|\vec{l}|^2}{2m_{D^{(*)}}} + i\varepsilon)(l^0 - q_1^0 - \frac{|\vec{l} - \vec{q}_1|}{2m_{D^*}} + i\varepsilon)} \\
 &\equiv G\epsilon_X^i \epsilon_{J/\psi}^j (3q_1^i q_1^j - |q_1|^2 \delta^{ij}) \mathcal{F}(M(J/\psi\pi), t) \text{I}(m_{D_1}, m_{D^{(*)}}, m_{D^*}, W, M(J/\psi\pi), m_\pi),
 \end{aligned} \tag{3}$$

where q_1 is the three momentum of the pion connected to the initial vector charmonium through the D_1 , G is the product of all the coupling constants from different vertices and I is the scalar three point loop function. Since our focus is on the dependencies of the loops on the external parameters in order to identify the singularity regions, we set $G = 1$ and only use the three-point scalar function I . This allows us to also investigate the effect of the width of the intermediate mesons. In any physical transition, pre-factors, which depend on the three-momentum q_1 , can distort the spectra to some extent, however, the general features of the amplitudes persist.

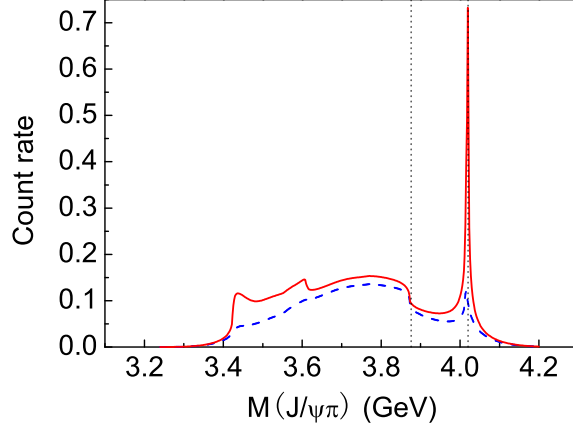


FIG. 4: The $J/\psi\pi$ invariant mass distribution at the center energy 4.43 GeV in the $J/\psi\pi\pi$ channel *with* (dashed) and *without* (solid) the width effects for the intermediate particles. The two vertical dotted lines denote the $\bar{D}D^*$ and \bar{D}^*D^* thresholds, respectively, from left to right.

A. Kinematics satisfying the two-cut condition in the vicinity of $\bar{D}D_1(2420)$

For the final state $J/\psi\pi\pi$, the kinematics in favor of the two-cut condition in the intermediate meson loops depends simultaneously on both the initial mass (W) as well as the invariant mass of $J/\psi\pi^\pm$ ($M(J/\psi\pi)$). A \bar{D}^*D cusp effect will be produced by diagrams (a) and (b) of Fig. 2. To illustrate this we show in the left panel of Fig. 3 the modulus of results for these diagrams in the W - $M(J/\psi\pi)$ -plane. For simplicity in Subsections II A-II C all intermediate mesons are treated as stable. The effect of their widths will be discussed later in Subsection III.

A singularity region can be identified where the transition amplitude is strongly enhanced and a pronounced cusp is expected around the $\bar{D}D^*$ threshold region for $4.28 < W < 4.31$ GeV. Unfortunately, in the preferred kinematic range there is no vector resonance, as can be read off from Fig. 1. Still, an energy scan of the e^+e^- system in this energy range would be very valuable. Interestingly, it should be noticed that there is still a visible enhancement even for $W \simeq 4.26$ GeV as shown in Fig. 3(a), due to the strong curvature of the contour lines. It is this enhancement that was discussed in Ref. [20].

In the diagrams of Fig. 2, the pion is radiated by the narrow $D_1(2420)$ which is assigned to be the mixed partner of the broad $D_1(2430)$ between the 1P_1 and 3P_1 states [36]. The spin symmetry demands that the 1P_1 state decays into $D^*\pi$ via a D -wave while the 3P_1 decays via an S -wave. Thus, it is the former that is to be identified with $D_1(2420)$, although some heavy quark symmetry breaking effects are expected and may result in mixings between these two configurations [36, 37].

Given that the narrow $D_1(2420)$ is to be dominated by the 1P_1 configuration, it will introduce a different momentum and angular dependence for the produced pion in comparison with the so-called “initial state pion emission (ISPE)” proposed in Refs. [15, 16]. Another distinct feature of the mechanism discussed here compared to the ISPE is its non-local character. A detailed measurement of the evolution of the $\pi\pi$ invariant mass spectra in terms of the initial e^+e^- c.m. energies could shed some light on the pion emission mechanism in the future. However, a proper theoretical treatment needs the inclusion of the $\pi\pi$ final state interactions, which goes beyond the scope of this paper and will be studied in a separated work.

B. Kinematics satisfying the two-cut condition in the vicinity of $\bar{D}^*D_1(2420)$

In the energy region where the $\bar{D}^*D_1(2420)$ intermediate state of diagrams (c) and (d) of Fig. 2 can be nearly on-shell, the two-cut condition is no longer satisfied for the $\bar{D}D_1(2420)$ intermediate state, which means that the $\bar{D}D^*$ cusp effect cannot be produced significantly in this energy region. However, in this kinematic region the \bar{D}^*D^* cusp can be produced. As shown by the right panel of Fig. 3, a strong enhancement can be expected in the invariant mass of $J/\psi\pi$ around the \bar{D}^*D^* threshold for $4.42 < W < 4.46$ GeV. This region also extends (though somewhat less

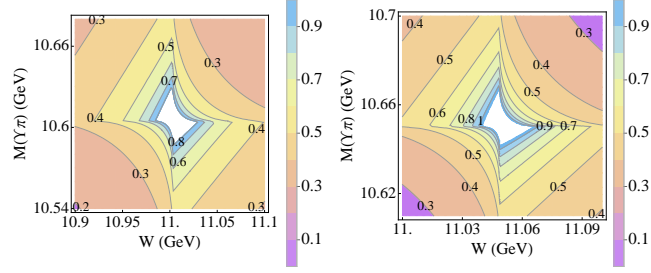


FIG. 5: The singularity region of $\bar{B}B^*$ (left panel) and \bar{B}^*B^* (right panel) without considering the widths of the intermediate mesons. The numbers have the same meanings as those in Fig. 3.

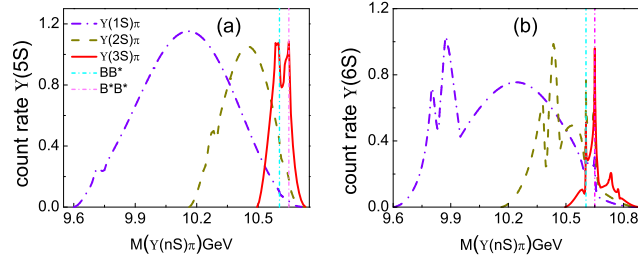


FIG. 6: The $\Upsilon(nS)\pi$ invariant mass distributions in (a) $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi\pi$ and (b) $\Upsilon(6S) \rightarrow \Upsilon(nS)\pi\pi$.

pronounced) even down to values of W as low as 4.38 GeV.

From Fig. 3 it becomes apparent that a simultaneous appearance of cusps at both the $\bar{D}D^*$ and \bar{D}^*D^* thresholds is not kinematically favored. This provides an explanation why there was no structure near the \bar{D}^*D^* threshold observed simultaneously with the discovery of the $Z_c(3900)$.

In addition, not only the strength but also the lineshape of the cusps change rather significantly when the initial energy changes. Such a behavior is very different from that of a resonance or a bound state, since their pole position is independent of the initial energy. We therefore expect that the dependence of the shape of a near-threshold structure on the initial energy contains direct information on the relative importance of the cusp and the resonance pole for a particular signal.

With the (red) solid line in Fig. 4, we show the invariant mass distribution of $J/\psi\pi$ at 4.43 GeV due to the processes listed in Fig. 2 — still with all particles assumed stable. This is the energy region where the two-cut condition is satisfied for \bar{D}^*D^* and a very pronounced cusp occurs at this threshold. Meanwhile, since for the $\bar{D}D^*$ threshold the two-cut condition is not satisfied for this initial energy, the corresponding cusp disappears, although it is accessible kinematically.

C. Kinematics satisfying the two-cut condition in the vicinity of $\bar{B}^*B_1(5721)$ and $\bar{B}^{(*)}B^*$

The above analysis can also be applied to the bottom sector. We present the plots showing the correlations between the initial mass and the invariant mass of $\Upsilon(3S)\pi$ in Fig. 5, where the cusps caused by the $\bar{B}B^*$ (left panel) or \bar{B}^*B^* (right panel) threshold can be easily identified. The singularity regions are very similar to those in the charm sector except that now there is a common kinematic region that allows those two cusps from the $\bar{B}B^*$ and \bar{B}^*B^* thresholds to appear simultaneously. The main reason is that $\Delta_B \equiv m_{B^*} - m_B = 46$ MeV is much smaller than $\Delta_D \equiv m_{D^*} - m_D = 142$ MeV.

In Fig. 6, we present the invariant mass distributions of the transitions $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi\pi$ (diagram (a)) and $\Upsilon(6S) \rightarrow \Upsilon(nS)\pi\pi$ (diagram (b)). It is interesting to see that the production of $\Upsilon(nS)\pi\pi$ does not satisfy the two-cut condition for the process $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi\pi$. Therefore, there are no obvious enhancements at the $\bar{B}B^*$ and \bar{B}^*B^* thresholds. In contrast, the $\Upsilon(6S)$ lies exactly in the singularity region which makes the two cusp peaks corresponding

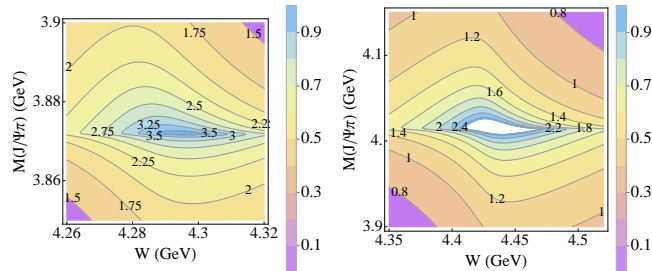


FIG. 7: The singularity region in the charm sector after considering the widths of the intermediate D_1 and D^* . Diagram (a) is for the $\bar{D}D^*$ singularity region and (b) is for \bar{D}^*D^* . The numbers have the same meanings as those in Fig. 3.

to the $\bar{B}B^*$ and \bar{B}^*B^* quite significant.

This result turns out to be important for understanding the nature of $Z_b(10610)$ and $Z'_b(10650)$ [4]. From the scenario studied in this work the structures called Z_b and Z'_b observed in $\Upsilon(5S) \rightarrow \Upsilon(1S, 2S)\pi\pi$ cannot be cusps but should result from explicit resonance poles, contrary to other claims in the literature [13, 15].

However, for the decay of $\Upsilon(6S)$ there should be simultaneously a large number of both $\bar{B}B^*$ as well as \bar{B}^*B^* pairs available. Therefore, if Z_b and Z'_b are hadron-hadron resonances and their existences are due to the non-perturbative $\bar{B}^{(*)}B^*$ interactions, their production should be favored in the decay of $\Upsilon(6S)$. For this scenario we therefore predict much stronger signals for these states in $\Upsilon(6S)$ than in $\Upsilon(5S)$ decays.

III. INFLUENCE OF THE WIDTH OF INTERMEDIATE STATES

Results for the singularity regions for the loops with the widths of the intermediate particles considered are presented in Fig. 7. Taking the singularity region in the charm sector as an example, we show the results after considering the width of the $D_1(2420)$ with $\Gamma_{D_1} = 27$ MeV [38] and $\Gamma_{D^*} = 190$ keV [39]. In comparison with the results shown in Fig. 3, we see that the cusp effects are smeared significantly for both the $\bar{D}D^*$ and \bar{D}^*D^* threshold. This becomes also clear from the dashed line in Fig. 4, where the $J/\psi\pi$ invariant mass distribution is shown for $W = 4.43$ GeV.

As shown by Fig. 7, above 4.32 GeV the structure at $\bar{D}D^*$ threshold is much more like a shoulder (see also dashed line in Fig. 4). This should be different from the enhancement caused by a pole structure.

IV. ANALYSIS OF THE SINGULARITY MECHANISM IN A P -WAVE TRANSITION

Recently the $Z_c(3900)$ signal is also reported in the $J/\psi\pi$ invariant mass distribution in the process $e^+e^- \rightarrow J/\psi\pi\pi$ at 4.17 GeV by an analysis using the CLEO-c data [3]. Although it is well below the first S -wave threshold (c.f. Fig. 1), there is a well established quarkonium, $\psi(4160)$, nearby and its dominant decay mode is $D^*\bar{D}^*$ [38, 40]. Thus, a meson loop analogous to the diagrams of Fig. 2 contains a P -wave vertex via the $\psi(4160)D^*\bar{D}^*$ interaction. Due to the centrifugal barrier cusps do not occur for partial waves higher than S -waves. Although for higher partial waves there still is a non-analyticity, it becomes visible in the derivative of amplitudes only [41]. However, the second part of the loop still produces a cusp, as can be seen in Fig. 8. Although the singularity region is now more limited in phase space, it still gives rise to some mild enhancement at 4.17 GeV. It implies that in order to explain the resonance signal observed by CLEO-c, an explicit resonance may be necessary, which turns out to be consistent with our findings in the higher energy region. Meanwhile, we anticipate that for $\psi(4040)$ even though it can give access to the $\bar{D}D^*$ cusp via its strong coupling to the $\bar{D}D^*$ channel [38, 40], the phase space would be extremely small and it remains to be seen if the $Z_c(3900)$ is observable at that low energies.

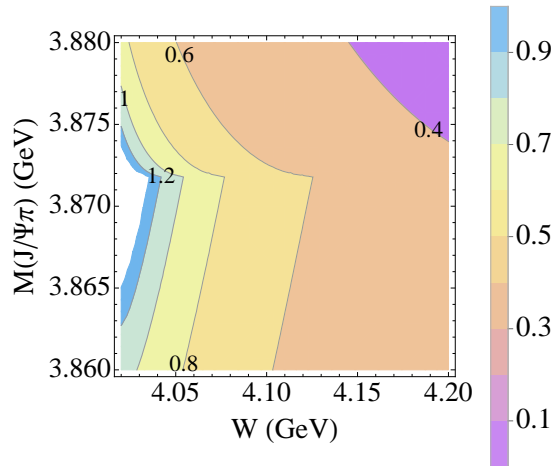


FIG. 8: The singularity region of heavy vector quarkonium decays into $J/\psi\pi\pi$ via the $D^*\bar{D}^*$ intermediate loops. The numbers have the same meanings as those in Fig. 3.

V. SUMMARY AND PERSPECTIVES

The above analysis has identified the kinematic regions in the heavy vector meson sectors where the relative S -wave heavy meson open thresholds may play an important role when a nearby vector state decays into a lighter quarkonium state plus two pions. It shows that there exist mass regions that can fulfill the two-cut condition such that the intermediate heavy meson loop can produce significant cusp effects. The clarification of the origin of the cusps and their evolutions with the initial masses would be important for a better understanding of these near-threshold enhancements recently observed in experiment, i.e. Z_b , Z'_b and Z_c etc. Based on our analysis, we argue, that these states should not be purely due to cusp effects if they can be observed out of the kinematics of the singularity regions identified in this work. We further argue that the dependence of these states on the initial energy for the production should reveal more information on whether they can be viewed as (predominantly) hadronic molecules or hadron-hadron resonances, or whether they should be viewed as more complicated structures.

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